1. The weight ratio of a rational Bézier curve is the largest divided by the smallest weight. For example, the weight ratio of a cubic Bézier curve with weights 8, 2, 3, 4 is 4. If you want to plot a rational Bézier curve using a fixed number of evenly-spaced line segments, the resulting plot will generally look smoother if the curve is reparameterized so that weight ratio is minimized. For example, a rational cubic Bézier curve with weights 8, 2, 3, 1 will not generally look as smooth as one with weights 2, 1, 3, 2. Given a rational cubic Bézier curve with weights 1, 4, 12, 27, reparameterize the curve so that the weight ratio is less than 2. (See Section 2.12, top of page 35 in the notes)

2. Convert the following power-basis polynomial into Bernstein form: (Section 3.3)

\[36t^4 - 72t^3 + 48t^2 - 12t + 1\]

3. Express the polynomial function

\[y = 36t^4 - 72t^3 + 48t^2 - 12t + 1\]

as an explicit Bézier curve (Section 2.14). Give the \((x, y)\) coordinates of the control points of the explicit Bézier curve.

4. Convert to a power-basis polynomial the following Bernstein basis polynomial: (Section 3.3)

\[4B_0^4(t) + 2B_1^4(t) - B_2^4(t) + 3B_3^4(t) + B_4^4(t)\]

5. Convert the following power-basis polynomial into a degree five Bernstein polynomial: (Section 3.3; write the polynomial as \(0t^5 + 0t^4 + 0t^3 + 10t^2 - 15t + 6\))

\[10t^2 - 15t + 6\]

6. A rational parametric curve is given by the equations

\[x(t) = \frac{t^3 + 3t^2 - 9t + 2}{-3t^3 + 3t^2 + 1}, \quad y(t) = \frac{2t^3 + 6t^2 - 3t + 3}{-3t^3 + 3t^2 + 1}.\]

Find the control points and weights for the equivalent rational Bézier curve. (Section 3.3. Convert the denominator to Bernstein basis to get the weights \(w_0B_0^3(t) + w_1B_1^3(t) + w_2B_2^3(t) + w_3B_3^3(t)\). Then convert the numerator polynomials to Bernstein form to get \(w_0x_0B_0^3(t) + w_1x_1B_1^3(t) + w_2x_2B_2^3(t) + w_3x_3B_3^3(t)\) and \(w_0y_0B_0^3(t) + w_1y_1B_1^3(t) + w_2y_2B_2^3(t) + w_3y_3B_3^3(t)\). Remember to divide the \(w_i x_i\) and \(w_i y_i\) by the weights to get the Cartesian coordinates \((x_i, y_i)\).

7. Draw the curves and their control polygons in problems 3 and 6 using CPLOT.

Hand in this homework at the beginning of class on 15 September.